Available online at www.sciencedirect.com
ScienceDirect
Journal of Applied Mathematics and Mechanics 70 (2006) 378–386
Journal of APPLIED MATHEMATICS AND MECHANICS
www.elsevier.com/locate/jappmathmech

# The effect of periodic surface pressure on a rectangular elastic plate floating on shallow water☆

I.V. Sturova

*Novosibirsk*

Received 30 March 2004

**Abstract**

The steady-state behaviour of a floating elastic plate of bounded dimensions acted upon by a localized external load is investigated using linear shallow water theory. In the case of a plate of arbitrary shape, the problem reduces to solving a system of boundary-value integral equations supplemented with differential relations for the free edge of the plate. Using a rectangular plate as the example, the effect of the frequency of the periodic actions and the positions at which they are applied on the amplitudes of the normal flexures of the plate and of the directional pattern of surface waves far from the plate is investigated. It is shown that waveguide properties occur for an elongated plate.


When determining the stability of large floating platforms of the floating-airport type (see Ref. 1, for example), an investigation of the effect of an unsteady external force on them is of interest. The simplest case of such action is a periodic pressure on the platform, which is usually simulated by a thin elastic plate.

The steady oscillations of a floating elastic plate when acted upon by a periodic load have been considered in a linear formulation in Ref. 2. In the planar case, a solution has been proposed for a girder plate of finite and semi-infinite length and, in the three-dimensional case, for a circular plate. Comparison of the solutions for shallow water and for a liquid of finite depth showed good agreement between them in the case of relatively low frequencies. The use of the shallow-water approximation and the method of boundary-value integral equations enables one to consider the problem of the action of periodic external pressures on a plate of arbitrary shape in a similar way to what was done independently in Refs. 3,4 in the case of a diffraction problem. The Green-Naghdi model was used in Ref. 3 which enables one to extend the domain of applicability of conventional shallow water theory somewhat to shorter waves.

In this paper, the linear hydroelastic problem of the action of periodic surface pressures on a plate of arbitrary shape is investigated for the case of a rectangular plate, since a rectangular shape for a floating platform is most frequently considered for practical application and, moreover, an elastic platform, in the form of a strip of constant width and infinite length, floating on shallow water can possess waveguide properties in the case of non-zero immersion.[5] An investigation of the effect of the finite length of a rectangular plate on the manifestation of waveguide properties is of interest.

☆ *Prikl. Mat. Mekh.* Vol. 70, No. 3, pp. 417–426, 2006.
*E-mail address:* sturova@hydro.nsc.ru.

0021-8928/$ – see front matter 
doi:10.1016/j.jappmathmech.2006.07.016

## 1. Formulation of the problem

A rectangular homogeneous elastic plate of length $2L$ and width $2B$ floats on the surface of a layer of an ideal incompressible liquid of depth $h$. The surface of the liquid not covered by the plate is free. We will denote the domain of the horizontal variables $x$ and $y$ occupied by the plate by $\Omega$ and the domain outside the plate by $\Omega_2$. The origin of coordinates is located at the centre of the domain $\Omega_1$ which is defined by the conditions $y \leq L$. Correspondingly, the domain $\Omega_2$ is defined by the conditions $|x| > B$, $|y| > L$.

We will assume that a normal pressure, periodic in time with frequency $\omega$, acts on the plate. We will investigate the oscillations of the liquid and the plate, caused by this pressure. The motion of the liquid is assumed to be potential and the velocity of the fluid particles and the flexure of the plate are assumed to be small.

$$p(x, y, t) = P(x, y)\exp(-i\omega t) \tag{1.1}$$

Assuming that the motions of the liquid and the plate are steady motions in time, we will correspondingly seek the velocity potentials $\phi_j\ (x, y, t)$ of the liquid in the domains $\Omega_j$ $(j = 1, 2)$ in the form

$$\phi_j(x, y, t) = \Phi_j(x, y)\exp(-i\omega t)$$

The normal flexure of the plate

$$w(x, y, t) = W(x, y)\exp(-i\omega t)$$

is described by the equation

$$D\Delta^2 W - \rho_1 h_1 \omega^2 W + g\rho W - i\omega\rho\Phi_1 = -P(x, y), \quad x, y \in \Omega_1 \tag{1.2}$$

where $D$ is the cylindrical stiffness of the plate, $h_1$ is its thickness, $\rho_1$ is the density of the material, $\rho$ is the liquid density, $g$ is the acceleration due to gravity and $\Delta \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$.

The relation

$$W = -i(h - d)\omega^{-1}\Delta\Phi_1, \quad x, y \in \Omega_1 \tag{1.3}$$

holds in the case of shallow water, where $d = \rho_1 h_1/\rho$ is the immersion of the plate. The velocity potential in the domain of free water $\Omega_2$ satisfies the equation

$$\Delta\Phi_2 + k_0^2\Phi_2 = 0, \quad x, y \in \Omega_2; \quad k_0 = \omega/\sqrt{gh} \tag{1.4}$$

The elevation of the free surface

$$\eta(x, y, t) = \zeta(x, y)\exp(-i\omega t)$$

is determined from the relation

$$\zeta = i\omega g^{-1}\Phi_2, \quad x, y \in \Omega_2$$

Matching conditions, which imply the continuity of the pressure and the mass flow,

$$\Phi_1 = \Phi_2, \quad \frac{\partial\Phi_1}{\partial n} = \frac{h}{h - d}\frac{\partial\Phi_2}{\partial n}, \quad x, y \in S \tag{1.5}$$

must be satisfied on the contour of the plate $S$, where $n$ is the direction of the normal to the contour $S$.

It is assumed that the plate edges are free, that is, the bending moment and the shearing force at the edges are equal to zero. In the case of a rectangular plate, the free edge conditions have the form (in the case of a plate of arbitrary shape, see 2,4, for example)

$$\Delta W = \nu_1\frac{\partial^2 W}{\partial s^2}, \quad \frac{\partial\Delta W}{\partial n} = -\nu_1\frac{\partial^3 W}{\partial n\partial s^2}, \quad x, y \in S; \quad \nu_1 = 1 - \nu \tag{1.6}$$

where $s$ is the arc coordinate of the contour $S$ and $\nu$ is Poisson's ratio of the plate.

The conditions for the bending moment of the concentrated shearing force to be compensated

$$\frac{\partial^2 W}{\partial x \partial y} = 0, \quad x = \pm B, \quad y = \pm L \tag{1.7}$$

must be satisfied at the corner points of the plate.

Far from the plate, it is necessary to take account of the radiation condition

$$\lim_{r \to \infty} \sqrt{r}\left(\frac{\partial}{\partial r} - ik_0\right)\Phi_2 = 0, \quad r = \sqrt{x^2 + y^2}$$

which implies that there are no incoming waves.

We next change to dimensionless variables, taking the depth of the tank $h$ as the scale of length and $\sqrt{h/g}$ as the time scale, that is

$$(\bar{x}, \bar{y}, \bar{r}, \overline{W}, \bar{\zeta}, \bar{L}, \bar{B}, \bar{d}) = \frac{1}{h}(x, y, r, W, \zeta, L, B, d)$$

$$\overline{\Phi}_j = \frac{1 - \bar{d}}{\sqrt{gh^3}}\Phi_j, \quad \bar{P} = \frac{P}{\rho g h}, \quad \bar{\omega} = \omega\sqrt{\frac{h}{g}}$$

## 2. Method of solution

Equations (1.2) and (1.4) in dimensionless variables have the form

$$\chi\Delta^3\overline{\Phi}_1 + \gamma\Delta\overline{\Phi}_1 + \tau\overline{\Phi}_1 = -i\bar{\omega}\bar{P}, \quad \bar{x}, \bar{y} \in \Omega_1 \tag{2.1}$$

$$\Delta\overline{\Phi}_2 + \bar{\omega}^2\overline{\Phi}_2 = 0, \quad \bar{x}, \bar{y} \in \Omega_2 \tag{2.2}$$

where

$$\chi = D/(\rho g h^4), \quad \gamma = 1 - \bar{d}\bar{\omega}^2, \quad \tau = \bar{\omega}^2/(1 - \bar{d})$$

We will seek the solution of Eq. (2.1) in the form (henceforth summation is carried out from $m=1$ to $m=3$)

$$\overline{\Phi}_1(\bar{x}, \bar{y}) = \Phi_0(\bar{x}, \bar{y}) + \sum \Psi_m(\bar{x}, \bar{y}) \tag{2.3}$$

The functions $\Psi_m(\bar{x}, \bar{y})$ $(m = 1, 2, 3)$ satisfy the equation

$$\Delta\Psi_m + \mu_m^2\Psi_m = 0 \tag{2.4}$$

and the quantities $\mu_m$ are the roots of the equation (for greater detail, see 3,4)

$$\chi\mu^6 + \gamma\mu^2 - \tau = 0 \tag{2.5}$$

We will denote the real positive root of this equation by $\mu_1$ and the two complex-conjugate roots, located in the first and fourth quadrants of the complex $\mu$ plane by $\mu_2$ and $\mu_3$ respectively.

The function $\Phi_0(\bar{x}, \bar{y})$ is the solution in dimensionless form of the problem of the action of a periodic pressure on an unbounded elastic plate in the shallow-water approximation.

For simplicity, we will next assume that the pressure $P(x, y)$ in (1.1) depends solely on the quantity $R = \sqrt{(x - x_0)^2 + (y - y_0)^2}$, where $x_0$ and $y_0$ are the coordinates of the centre of the domain of application of the external pressure, that is,

$$P(x, y) = a\rho g f(R)$$

Here $a$ is a factor which has the dimension of time and the function $f(R)$ is dimensionless.

The function $\Phi_0(\bar{x}, \bar{y})$ has the form (for more detail, see 2)

$$\Phi_0 = -\frac{i\bar{a}\bar{\omega}}{2}\left[\frac{1}{\pi}\text{p.v.}\int_0^\infty \frac{\tilde{f}(k)J_0(k\bar{R})dk}{Q(k)[\bar{\omega}^2 - S(k)]} - \frac{i\tilde{f}(\mu_1)J_0(\mu_1\bar{R})}{Q(\mu_1)S'(\mu_1)}\right]$$

$$\tilde{f}(k) = 2\pi\int_0^\infty \bar{R}f(\bar{R})J_0(k\bar{R})d\bar{R}, \quad Q(k) = \bar{d}k + \frac{1}{k(1-\bar{d})}$$

$$S(k) = \frac{k(1+\chi k^4)}{Q(k)}, \quad S'(\mu_1) = \left.\frac{dS}{dk}\right|_{k=\mu_1}, \quad \bar{a} = \frac{a}{h}, \quad \bar{R} = \frac{R}{h}$$

where $\tilde{f}(k)$ is a double Fourier transform of the function $f(\bar{R})$, $J_0(\cdot)$ is a zero-order Bessel function of the first kind, p. v. denotes an integral in the sense of the principal value and $\mu_1$ is the real positive root of the equation $\bar{\omega}^2 = S(k)$ which is identical to Eq. (2.5).

Equations (2.2) and (2.4) are Helmholtz equations. In the general case, the corresponding Green's function $G(\mathbf{r}, \mathbf{r}_1; k)$ satisfies the equation

$$\Delta G + k^2 G = 2\pi\delta(\mathbf{r} - \mathbf{r}_1), \quad \mathbf{r} = (\bar{x}, \bar{y}), \quad \mathbf{r}_1 = (\bar{x}_1, \bar{y}_1)$$

where $\delta(\cdot)$ is the Dirac delta-function. The requirement that the radiation condition is satisfied in the far field leads to the representation

$$G(\mathbf{r}, \mathbf{r}_1; k) = -\frac{i\pi}{2}H_0^{(1)}(kR_1), \quad R_1 = \sqrt{(\bar{x}-\bar{x}_1)^2 + (\bar{y}-\bar{y}_1)^2} \tag{2.6}$$

where $H_0^{(1)}(\cdot)$ is a zero order Hankel function of the first kind. In the domain $\Omega_1$, Green's function can be used in the form (2.6) and it can also be expressed in terms of other cylindrical functions.[3]

By using Green's theorem in the domain $\Omega_1$, we obtain

$$\varepsilon_1\Psi_m(\mathbf{r}) + \frac{1}{\pi}\int_S\left[\Psi_m(\mathbf{r}_1)\frac{\partial G}{\partial n}(\mathbf{r}, \mathbf{r}_1; \mu_m) - G(\mathbf{r}, \mathbf{r}_1; \mu_m)\frac{\partial \Psi_m}{\partial n}(\mathbf{r}_1)\right]ds = 0, \quad \bar{x}, \bar{y} \in \Omega_1 \tag{2.7}$$

where $\varepsilon_1 = 2$ if the point $\mathbf{r}$ is within $S$, $\varepsilon_1 = 1$ if $\mathbf{r}$ is on a smooth segment of $S$ and $\varepsilon_1 = 1/2$ if $\mathbf{r}$ is a corner point of the rectangular domain $\Omega_1$.

A similar integral relation holds in the domain $\Omega_2$

$$\varepsilon_2\Phi_2(\mathbf{r}) = \frac{1}{\pi}\int_S\left[\Phi_2(\mathbf{r}_1)\frac{\partial G}{\partial n}(\mathbf{r}, \mathbf{r}_1; \bar{\omega}) - G(\mathbf{r}, \mathbf{r}_1, \bar{\omega})\frac{\partial \Phi_2}{\partial n}(\mathbf{r}_1)\right]ds, \quad \bar{x}, \bar{y} \in \Omega_2 \tag{2.8}$$

where $\varepsilon_2 = 2$ if the point $\mathbf{r}$ is outside the contour $S$, $\varepsilon_2 = 1$ if $\mathbf{r}$ is in a smooth segment of $S$ and $\varepsilon_2 = 3/2$ at the corner points.

In order to solve the initial problem, it is necessary to determine the values of $\Psi_m(\mathbf{r})$ and $\partial\Psi_m(\mathbf{r})/\partial n$ ($m = 1, 2, 3$) on the contour $S$. By using points $\mathbf{r}$, belonging to $S$, we obtain a system of four integral equations, the first three of which are Eq. (2.7) when $m = 1, 2$, and 3 respectively, while the fourth equation, according to relations (1.5), (2.3) and (2.8) has the form

$$\begin{aligned}\varepsilon_2[\Phi_0(\mathbf{r}) + \sum\Psi_m(\mathbf{r})] &= \frac{1}{\pi}\int_S\left\{\frac{\partial G}{\partial n}(\mathbf{r}, \mathbf{r}_1; \bar{\omega})[\Phi_0(\mathbf{r}_1) + \sum\Psi_m(\mathbf{r}_1)] - \right.\\ &\left. -(1-\bar{d})G(\mathbf{r}, \mathbf{r}_1; \bar{\omega})\left[\frac{\partial \Phi_0}{\partial n}(\mathbf{r}_1) + \sum\frac{\partial \Psi_m}{\partial n}(\mathbf{r}_1)\right]\right\}ds, \quad \bar{x}, \bar{y} \in S\end{aligned} \tag{2.9}$$

Two supplementary differential equations are obtained from the conditions of a free edge (1.6) which, according to relations (1.3), (2.3) and (2.4), can be written in the form

$$\sum \mu_m^2\left(\mu_m^2\Psi_m + \nu_1\frac{\partial^2\Psi_m}{\partial s^2}\right) = \Delta\left(\nu_1\frac{\partial^2\Phi_0}{\partial s^2} - \Delta\Phi_0\right) \tag{2.10}$$

$$\sum \mu_m^2\frac{\partial}{\partial n}\left(\mu_m^2\Psi_m - \nu_1\frac{\partial^2\Psi_m}{\partial s^2}\right) = -\Delta\frac{\partial}{\partial n}\left(\Delta\Phi_0 + \nu_1\frac{\partial^2\Phi_0}{\partial s^2}\right), \quad \bar{x}, \bar{y} \in S \tag{2.11}$$

By condition (1.7), the equality

$$\sum \mu_m^2\frac{\partial^2\Psi_m}{\partial\bar{x}\partial\bar{y}} = \Delta\frac{\partial^2\Phi_0}{\partial\bar{x}\partial\bar{y}}, \quad \bar{x} = \pm\bar{B}, \quad y = \pm\bar{L}$$

must be satisfied at the corner points.

After determining the boundary-values of $\Psi_m$ and $\partial\Psi_m/\partial n$ ($m = 1, 2, 3$) on the contour $S$, the buckling of the plate

$$\bar{W} = \frac{i}{\bar{\omega}}\left(\sum \mu_m^2\Psi_m - \Delta\Phi_0\right), \quad \bar{x}, \bar{y} \in \Omega_1$$

can be calculated.

Using the asymptotic representation of Green's function (2.6) for the domain $\Omega_2$ in the far field

$$G(\mathbf{r}, \mathbf{r}_1; \bar{\omega}) \approx -\sqrt{\frac{\pi}{2\bar{\omega}\bar{r}}}\exp\left\{i\left[\bar{\omega}(\bar{r} - \bar{x}_1\cos\theta - \bar{y}_1\sin\theta) + \frac{\pi}{4}\right]\right\}, \quad \bar{r} \to \infty$$

the amplitudes of the surface waves far from the plate can be expressed in terms of Kochin's function

$$H(\bar{\omega}, \theta) = \int_S\left[\frac{\partial\bar{\Phi}_2}{\partial n} + i\bar{\omega}\bar{\Phi}_2(n_x\cos\theta + n_y\sin\theta)\right]\exp[-i\bar{\omega}(\bar{x}_1\cos\theta + \bar{y}_1\sin\theta)]ds$$

We have

$$|\bar{\zeta}| = \sqrt{\frac{\bar{\omega}}{8\pi\bar{r}}}\frac{|H(\bar{\omega}, \theta)|}{1 - \bar{d}}, \quad \theta = \operatorname{arctg}\frac{\bar{y}}{\bar{x}}$$

where $n_x$ and $n_y$ are the components of the vector of the outward normal to the contour $S$ at the point $\bar{x}_1$, $\bar{y}_1$.

In the numerical solution of Eqs. (2.7), (2.9)–(2.11), segments of the contour $S$, parallel to the $x$ and $y$ axes, are subdivided into $N_x$ and $N_y$ equal segments respectively. The numerical method of solution has been described in Ref. 4 and an alternative method can be found in Ref. 3.

## 3. The waveguide properties of an elastic strip

It was shown in Ref. 5 that, for an elastic platform in the form of a strip of finite width and infinite length which floats on shallow water, a natural waveguide mode can exist which propagates along the strip and decays exponentially far from it. The necessary condition for a waveguide mode to exist is that there is a non-zero immersion of the platform.

We will now briefly describe the solution of the problem of determining the characteristics of waves trapped by an elastic strip of width $2B$. It is necessary to find a non-trivial solution of the homogeneous equations for the corresponding velocity potentials, which follow from relations (1.2)–(1.4)

$$D\Delta^3\Phi_1 + (g\rho - \rho_1 h_1\omega^2)\Delta\Phi_1 + \frac{\rho\omega^2}{h - d}\Phi_1 = 0, \quad |x| \le B, \quad |y| < \infty \tag{3.1}$$

$$\Delta\Phi_2 + k_0^2\Phi_2 = 0, \quad |x| > B, \quad |y| < \infty \tag{3.2}$$

with the free edge conditions, which follow from (1.3) and (1.6)

$$\Delta^2\Phi_1 = \nu_1\Delta\frac{\partial^2\Phi_1}{\partial y^2}, \quad \Delta^2\frac{\partial\Phi_1}{\partial x} = -\nu_1\Delta\frac{\partial^3\Phi_1}{\partial x\partial y^2}, \quad x = \pm B, \quad |y| < \infty \tag{3.3}$$

and the matching conditions, which are analogous to (1.5),

$$\Phi_1 = \Phi_2, \quad \frac{\partial\Phi_1}{\partial x} = \frac{h}{h-d}\frac{\partial\Phi_2}{\partial x}, \quad x = \pm B, \quad |y| < \infty \tag{3.4}$$

We will seek a solution of Eqs. (3.1) and (3.2) in the form

$$\Phi_j(x, y) = \Psi_j(x)\exp(i\lambda y), \quad j = 1, 2 \tag{3.5}$$

In the case of waves trapped by the elastic strip, it is necessary that the following conditions should be satisfied in the far field

$$\Psi_2 \to 0, \quad |x| \to \infty$$

according to which the solution for $\Psi_2$ $(x)$, when account is taken of relations (3.2) and (3.5), has the form

$$\Psi_2 = \begin{cases}\alpha_+\exp(-\beta x), & x > B\\ \alpha_-\exp(\beta x), & x < -B\end{cases}; \quad \beta = \sqrt{\lambda^2 - k_0^2}$$

where $\alpha_\pm$ are unknown constants. The value of $\beta$ must be real and positive and, consequently, $\lambda > k_0$. This inequality implies that normal oscillations of the elastic strip cannot be excited by the free surface waves since for these waves we always have $\lambda \le k_0$ (see Ref. 6, for example).

It was shown in Ref. 5 that the normal oscillations of an elastic strip are symmetrical about to the $y$ axis. By virtue of this condition, the solution for the function $\psi_1(x)$ can be written in the form

$$\Psi_1(x) = \sum c_m \operatorname{ch}(\sigma_m x), \quad |x| \le B \tag{3.6}$$

where $c_m$ are unknown constants and the values of $\sigma_m$ are determined from an equation which is analogous to (2.5) after substituting (3.5), into Eq. (3.1), taking account of expression (3.6). From the boundary conditions (3.3) and the matching conditions (3.4) we obtain a homogeneous system of linear fourth-order algebraic equations for determining $\alpha_+$, $c_1$, $c_2$, $c_3$. Those values of $\omega$ for which the determinant of this system vanishes are called the natural frequencies of the elastic strip.

## 4. Results of calculations

The numerical results presented below were obtained for two versions of the choice of elastic strip parameters:

1) a laboratory model of a floating airport[4]

$$D = 1.093\cdot 10^3\,\text{kg.m}^2/\text{s}^2, \quad h = 25\,\text{cm}, \quad 2L = 15\,\text{m}, \quad 2B = 3\,\text{m}, \quad d = 1.25\,\text{cm},$$
$$\chi = 28.52, \quad \bar{d} = 0.05$$

2) a design for a real floating airport[3]

$$D = 1.96\cdot 10^{11}\,\text{kg.m}^2/\text{s}^2, \quad h = 50\,\text{m}, \quad 2L = 5\,\text{km}, \quad 2B = 1\,\text{km}, \quad d = 5\,\text{m},$$
$$\chi = 3.197, \quad \bar{d} = 0.1$$

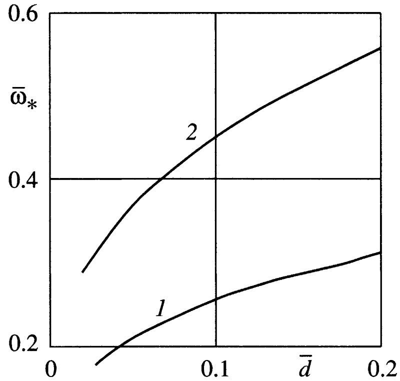


Fig. 1.

In both cases, the liquid density $\rho = 10^3$ kg/m$^3$ and Poisson's ratio of the plate $\nu = 0.3$. First we will present the characteristics of the natural frequencies for an elastic strip of width $2B$ and the values of $D$, $h$ and $d$ indicated for the two versions. As was shown in Ref. 5, a waveguide mode only exists when $d > 0$ and in the frequency range $0 < \omega < \omega_*$. In the process, the corresponding wave number $\lambda$ only very slightly exceeds the value $\omega/\sqrt{gh}$ (see Ref. 5, Fig. 1). The values of $\omega_*$ are: $1.312\,\mathrm{s}^{-1}$ ($\bar{\omega}_* = 0.209$) for version 1 and $0.199\,\mathrm{s}^{-1}$ $\bar{\omega}_* = 0.450$ for version 2. The relative excess of $\lambda$ above the value $\omega/\sqrt{gh}$ was no greater than 2%.

Graphs of $\bar{\omega}_*$ against the dimensionless value of the immersion of the elastic strip $\bar{d}$ for the parameters corresponding to version 1 (curve *1*) and version 2 (curve *2*) are shown in Fig. 1. When the immersion increases, the range of frequencies for which a waveguide mode exists expands monotonically.

The action of a periodic pressure on a rectangular plate was investigated for the function

$$f(R) = [1-(R/l)^2], \quad R<l; \quad f(R) = 0, \quad R>l; \quad \tilde{f}(k) = 4\pi J_2(kl)/l^2$$

In all the calculations presented below, the radius of the domain of application of the pressure was equal to $l = 2h$ and the centre of this domain was located on the central line of the plate ($x_0 = 0$).

The isolines of the amplitudes of the normal flexures of the plate are shown in Fig. 2 for version 2 and $\bar{y}_0 = -25$ when $\bar{\omega} = 0.3$ (the upper half of Fig. 2) and $\bar{\omega} = 0.6$ (the lower half). The isolines of the function $10^2 \times |W|/a$ are drawn out with levels from 1 to 15 with a step size of 2. The values of $|W|/a$ at the centre of the domain where pressure is applied are equal to 0.354 and 0.285 when $\bar{\omega} = 0.3$ and $\bar{\omega} = 0.6$ respectively. The calculations were carried out for $N_x = 6$ and $N_y = 30$ and a further increase in these parameters hardly changed the results. The same values of $N_x$ and $N_y$ were used below in all the calculations carried out for a ratio of the sides of a rectangular plate $L/B = 5$.

The effect of the site where the external pressure was applied on the behaviour of the plate is shown in Fig. 3 for version 1 when $\bar{\omega} = 0.4$. The centre of the domain where the pressure was applied is located at the point $y_0 = 0$ and $\bar{y}_0 = -15$ for the upper and lower halves of the figure respectively. The isolines of the function $10^2 \times |W|/a$ are drawn out from 2 to 11 with a step size of 1.

More detailed information concerning the behaviour of $|W|/a$ in the centre line $x = 0$ (the solid curves) and at the edge of the plate $|x| = B$ (the dashed curves) is shown in Fig. 4 for $y_0 = 0$ (curves *1*) and $\bar{y}_0 = -15$ (curves *2*).

It is clear from an analysis of the results presented in Figs. 2 to 4 that, when $\bar{\omega} > \bar{\omega}_*$, the domains of significant buckling of the plate correspond not only to the central part of the domain where the pressure is applied but also to

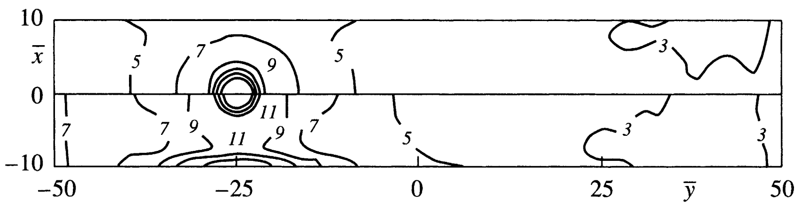


Fig. 2.

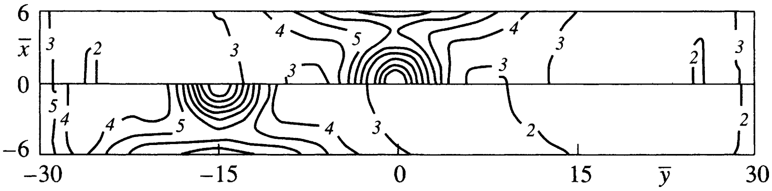


Fig. 3.

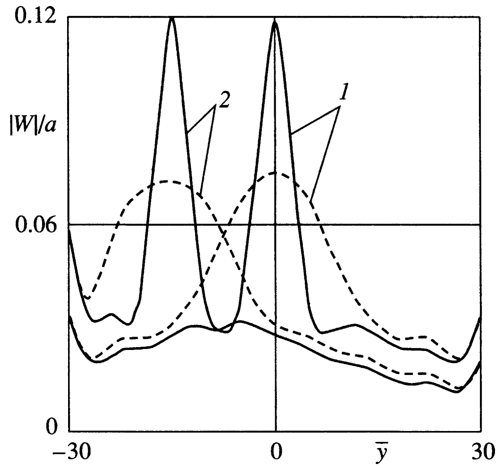


Fig. 4.

the plate edges. At the same time, the pattern of the amplitude distribution of the oscillations of the bucklings in the neighbourhood of the domain where the pressure is applied up to the closest edges of the plate depend only slightly on the value of $y_0$. Near the plate edges, the amplitudes of its bucklings barely change in the transverse direction.

The behaviour of the surface waves far from the elastic plate is conveniently described using a beam pattern. The dependence of $Q = |H(\bar{\omega}, \theta)|/\sqrt{8\pi\bar{\omega}}$ on the angle θ is represented in polar coordinates in Fig. 5. Here, the left-hand halves of Fig. 5, *a* and *b* correspond to plates with a ratio of the sides *L*/*B* = 5 and the right-hand halves correspond to longer plates with *L*/*B* = 7. Calculations were carried out for *L*/*B* = 7 when $N_x = 6$, $N_y = 42$. The left-hand side of Fig. 5, *a* was constructed for version 1 when $\bar{y}_0 = -15$ and the right-hand side with the same values of *D*, *h*, *d* and *B* but for *L* = 21 m and $\bar{y}_0 = -27$. The solid curves in Fig. 5, *a* correspond to a frequency $\bar{\omega} = 0.1$, the dashed curves to a frequency $\bar{\omega} = \bar{\omega}_* = 0.209$, and the dot-dash curves to $\bar{\omega} = 0.4$. The left-hand side of Fig. 5, *b* corresponds to version 2 when $\bar{y}_0 = -25$ and the right-hand side to the same values of *D*, *h*, *d* and *B* but for *L* = 7 km and $\bar{y}_0 = -45$. The solid

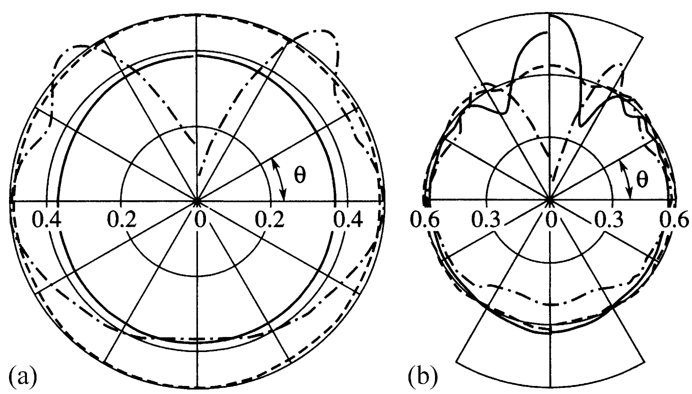


Fig. 5.

curves in Fig. 5, *b* correspond to the frequency $\bar{\omega} = 0.3$, the dashed curves to $\bar{\omega} = \bar{\omega}_* = 0.45$ and the dot-dash curves to $\bar{\omega} = 0.6$.

It is clear that the forms of the beam patterns for the frequencies $\bar{\omega} < \bar{\omega}_*$ and $\bar{\omega} > \bar{\omega}_*$ are different. For a sufficiently low limiting value of the waveguide mode (Fig. 5, *a*), the beam pattern when $\bar{\omega} \leq \bar{\omega}_*$ is close to a circle. This means that the amplitudes of the surface waves are practically equal in all directions. However, when $\bar{\omega}_*$ is increased, the waveguide character of the plate shows itself to a greater extent: surface waves of the greatest amplitude propagate in the direction of a long side of the plate, that is, along the *y*-axis. This effect is reinforced as the length of the plate is increased (see Fig. 5, *b*).

In the case of external oscillations with a frequency $\bar{\omega} > \bar{\omega}_*$, a reduction in the scattering of surface waves is observed along the plate, particularly in the direction of its far end with respect to the pressure domain.

The above results show that the proposed method is an effective technique for investigating the behaviour of an elastic plate floating on shallow water under the action of an external low-frequency periodic load. The method can be used in the case of a bounded plate of arbitrary shape. As was shown earlier in the case of a circular plate,[2] the finite dimensions of a plate have a considerable effect on the characteristics of its oscillations under the action of a periodic load. An elongated rectangular plate can possess waveguide properties just like a floating elastic strip of infinite length.

## Acknowledgements

This research was carried out within the framework of the “State Support for Leading Scientific Schools” programme (Nsh-902.2003.1) with the support of the Russian Foundation for Basic Research (02-01-00739).

## References

1. *Hydroelasticity in Marine Technology: Proc. 3rd Intern. Conf.* Ed. R.E. Taylor. Oxford, UK, 2003, 328 p.
2. Sturova IV. The action of periodic surface pressures on a floating elastic platform. *Prikl Mat Mekh* 2002;**66**(1):75–86.
3. Ertekin RC, Kim JW. Hydroelastic response of a floating mat-type structure in oblique, shallow-water waves. *J Ship Res* 1999;**43**(4):241–54.
4. Sturova IV. Diffraction of surface waves on an elastic platform floating in shallow water. *Prikl Mat Mekh* 2001;**65**(1):114–22.
5. Tkacheva LA. Natural vibrations of an elastic platform floating on shallow water. *Zh Prikl Mekh Tekh Fiz* 2000;**41**(1):173–81.
6. Sturova IV. Oblique impact of surface waves on an elastic strip. *Zh Prikl Mekh Tekhn Fiz* 1999;**40**(4):62–8.

*Translated by* E.L.S.